For the two (2) items that follow:

Consider the function

\[ f(x) = \frac{a^{[x]+x} - 1}{[x] + x} \]

where \([\cdot]\) denotes the greatest integer function.

1. What is \( \lim_{x \to 0^+} f(x) \) equal to?
   
   \( (a) \) 1  
   \( (b) \) \ln a  
   \( (c) \) \(1 - a^{-1}\)  
   \( (d) \) Limit does not exist

2. What is \( \lim_{x \to 0^-} f(x) \) equal to?
   
   \( (a) \) 0  
   \( (b) \) \ln a  
   \( (c) \) \(1 - a^{-1}\)  
   \( (d) \) Limit does not exist

For the next two (2) items that follow:

Let \( z_1, z_2 \) and \( z_3 \) be non-zero complex numbers satisfying \( z^2 = iz \), where \( i = \sqrt{-1} \).

3. What is \( z_1 + z_2 + z_3 \) equal to?
   
   \( (a) \) \( i \)  
   \( (b) \) \(-i\)  
   \( (c) \) 0  
   \( (d) \) 1

4. Consider the following statements:
   
   1. \( z_1z_2z_3 \) is purely imaginary.
   2. \( z_1z_2 + z_2z_3 + z_3z_1 \) is purely real.

   Which of the above statements is/are correct?
   
   \( (a) \) 1 only  
   \( (b) \) 2 only  
   \( (c) \) Both 1 and 2  
   \( (d) \) Neither 1 nor 2

For the next two (2) items that follow:

Given that \( \log_x y, \log_y x, \log_z y \) are in GP, \( xyz = 64 \) and \( x^3, y^3, z^3 \) are in AP.

5. Which one of the following is correct?
   
   \( x, y \) and \( z \) are
   
   \( (a) \) in AP only  
   \( (b) \) in GP only  
   \( (c) \) in both AP and GP  
   \( (d) \) neither in AP nor in GP
6. Which one of the following is correct?

- $xy, yz$ and $zx$ are
  - (a) in AP only
  - (b) in GP only
  - (c) in both AP and GP
  - (d) neither in AP nor in GP

For the next two (2) items that follow:

Let $z$ be a complex number satisfying

\[ \frac{|z-4|}{|z-8|} = 1 \text{ and } \frac{|z|}{|z-2|} = \frac{3}{2} \]

7. What is $|z|$ equal to?

- (a) 6
- (b) 12
- (c) 18
- (d) 36

8. What is $|\frac{z-6}{z+6}|$ equal to?

- (a) 3
- (b) 2
- (c) 1
- (d) 0

For the next two (2) items that follow:

A function $f(x)$ is defined as follows:

\[ f(x) = \begin{cases} 
  x+\pi & \text{for } x \in [-\pi, 0) \\
  \pi \cos x & \text{for } x \in \left[0, \frac{\pi}{2}\right] \\
  (x-\frac{\pi}{2})^2 & \text{for } x \in \left(\frac{\pi}{2}, \pi\right] 
\end{cases} \]

9. Consider the following statements:

1. The function $f(x)$ is continuous at $x = 0$.
2. The function $f(x)$ is continuous at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

10. Consider the following statements:

1. The function $f(x)$ is differentiable at $x = 0$.
2. The function $f(x)$ is differentiable at $x = \frac{\pi}{2}$.

Which of the above statements is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2
For the next two (2) items that follow:

Let \( a \) and \( \beta \) \((a < \beta)\) be the roots of the equation \( x^2 + bx + c = 0 \), where \( b > 0 \) and \( c < 0 \).

11. Consider the following:
   
   1. \( \beta < -\alpha \)
   2. \( \beta < |\alpha| \)

Which of the above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

12. Consider the following:

1. \( x^2 + bx + c = 0 \) \((\alpha, \beta)\)

Which of the above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

13. What is the value of \( AC^2 - BD^2 \)?

   (a) 25
   (b) 30
   (c) 36
   (d) 40

14. What is the point of intersection of the diagonals?

   (a) \( \left(\frac{7}{2}, 4\right) \)
   (b) \( (3, 4) \)

15. What is the area of the parallelogram?

   (a) \( \frac{7}{2} \) square units
   (b) 4 square units
   (c) \( \frac{11}{2} \) square units
   (d) 7 square units
16. Let $f(x)$ and $g(x)$ be twice differentiable functions on $[0, 2]$ satisfying $f''(x) = g''(x)$, $f'(1) = 4$, $g'(1) = 6$, $f(2) = 3$ and $g(2) = 9$. Then what is $f(x) - g(x)$ at $x = 4$ equal to?

(a) $-10$
(b) $-6$
(c) $-4$
(d) $2$

For the next two (2) items that follow:

Consider the function

$$f(x) = \begin{vmatrix} x^3 & \sin x & \cos x \\ 6 & -1 & 0 \\ p & p^2 & p^3 \end{vmatrix}$$

where $p$ is a constant.

19. What is the value of $f'(0)$?

(a) $p^3$
(b) $3p^3$
(c) $6p^3$
(d) $-6p^3$

20. What is the value of $p$ for which $f''(0) = 0$?

(a) $-\frac{1}{6}$ or 0
(b) $-1$ or 0
(c) $-\frac{1}{6}$ or 1
(d) $-1$ or 1

For the next two (2) items that follow:

Consider a triangle $ABC$ in which

$$\cos A + \cos B + \cos C = \sqrt{3} \sin \frac{\pi}{3}$$

21. What is the value of $\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$?

(a) $\frac{1}{2}$
(b) $\frac{1}{4}$
(c) $\frac{1}{8}$
(d) $\frac{1}{16}$
22. What is the value of \( \cos \left( \frac{A+B}{2} \right) \cos \left( \frac{B+C}{2} \right) \cos \left( \frac{C+A}{2} \right) \)?

(a) \( \frac{1}{4} \)  \( \quad \) (1, 3)
(b) \( \frac{1}{2} \)  \( \quad \) (7, -4)
(c) \( \frac{1}{16} \)  \( \quad \) (4, 4)
(d) None of the above

For the next two (2) items that follow:

Given that \( \tan \alpha \) and \( \tan \beta \) are the roots of the equation \( x^2 + bx + c = 0 \) with \( b \neq 0 \).

23. What is \( \tan(\alpha + \beta) \) equal to?

(a) \( b(c-1) \)
(b) \( c(b-1) \)
(c) \( c(b-1)^{-1} \)
(d) \( b(c-1)^{-1} \)

24. What is \( \sin(\alpha + \beta) \sec\alpha \sec\beta \) equal to?

(a) \( b \)
(b) \( -b \)
(c) \( c \)
(d) \( -c \)

For the next two (2) items that follow:

Consider the two circles
\( (x-1)^2 + (y-3)^2 = r^2 \) and
\( x^2 + y^2 - 8x + 2y + 8 = 0 \)

25. What is the distance between the centres of the two circles?

(a) 5 units  \( \quad \) (1, 3)
(b) 6 units  \( \quad \) (4, -1)
(c) 8 units  \( \quad \) \( \sqrt{9} + 16 \)
(d) 10 units

26. If the circles intersect at two distinct points, then which one of the following is correct?

(a) \( r = 1 \)
(b) \( 1 < r < 2 \)
(c) \( r = 2 \)
(d) \( 2 < r < 8 \)

For the next two (2) items that follow:

Consider the two lines
\( x + y + 1 = 0 \) and \( 3x + 2y + 1 = 0 \)

27. What is the equation of the line passing through the point of intersection of the given lines and parallel to \( x \)-axis?

(a) \( y + 1 = 0 \)
(b) \( y - 1 = 0 \)
(c) \( y - 2 = 0 \)
(d) \( y + 2 = 0 \)
28. What is the equation of the line passing through the point of intersection of the given lines and parallel to $y$-axis?

(a) $x + 1 = 0$

$\therefore$ $x - 1 = 0$

(c) $x - 2 = 0$

(d) $x + 2 = 0$

For the next two (2) items that follow:

Consider the equation

$$k \sin x + \cos 2x = 2k - 7$$

29. If the equation possesses solution, then what is the minimum value of $k$?

(a) 1

(b) 2

(c) 4

(d) 6

30. If the equation possesses solution, then what is the maximum value of $k$?

(a) 1

(b) 2

(c) 4

(d) 6

For the next two (2) items that follow:

Consider the functions

$$f(x) = xg(x) \quad \text{and} \quad g(x) = \left[ \frac{1}{x} \right]$$

where $\lfloor \cdot \rfloor$ is the greatest integer function.

31. What is $\int_{a}^{b} g(x) \, dx$ equal to?

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{5}{18}$

(d) $\frac{5}{36}$

32. What is $\int_{a}^{b} f(x) \, dx$ equal to?

(a) $\frac{37}{72}$

(b) $\frac{7}{3}$

(c) $\frac{17}{72}$

(d) $\frac{37}{144}$

For the next five (5) items that follow:

Consider the function

$$f(x) = |x - 1| + x^2$$

where $x \in \mathbb{R}$.

33. Which one of the following statements is correct?

(a) $f(x)$ is continuous but not differentiable at $x = 0$

(b) $f(x)$ is continuous but not differentiable at $x = 1$

(c) $f(x)$ is differentiable at $x = 1$

(d) $f(x)$ is not differentiable at $x = 0$ and $x = 1$
34. Which one of the following statements is correct?
   (a) \( f(x) \) is increasing in \((-\infty, \frac{1}{2})\) and decreasing in \(\left(\frac{1}{2}, \infty\right)\)
   (b) \( f(x) \) is decreasing in \((-\infty, \frac{1}{2})\) and increasing in \(\left(\frac{1}{2}, \infty\right)\)
   (c) \( f(x) \) is increasing in \((-\infty, 1)\) and decreasing in \((1, \infty)\)
   (d) \( f(x) \) is decreasing in \((-\infty, 1)\) and increasing in \((1, \infty)\)

35. Which one of the following statements is correct?
   (a) \( f(x) \) has local minima at more than one point in \((-\infty, \infty)\)
   (b) \( f(x) \) has local maxima at more than one point in \((-\infty, \infty)\)
   (c) \( f(x) \) has local minimum at one point only in \((-\infty, \infty)\)
   (d) \( f(x) \) has neither maxima nor minima in \((-\infty, \infty)\)

36. What is the area of the region bounded by \(x\)-axis, the curve \( y = f(x) \) and the two ordinates \( x = \frac{1}{2} \) and \( x = 1 \)?
   (a) \( \frac{5}{12} \) square unit
   (b) \( \frac{5}{6} \) square unit
   (c) \( \frac{7}{6} \) square units
   (d) 2 square units

37. What is the area of the region bounded by \(x\)-axis, the curve \( y = f(x) \) and the two ordinates \( x = 1 \) and \( x = \frac{3}{2} \)?
   (a) \( \frac{5}{12} \) square unit
   (b) \( \frac{7}{12} \) square unit
   (c) \( \frac{2}{3} \) square unit
   (d) \( \frac{11}{12} \) square unit

For the next two (2) items that follow:

Given that
\[
a_n = \int_0^\pi \frac{\sin^2 \left(\frac{(n+1)x}{2}\right)}{1-\cos^2 x} \, dx
\]

38. Consider the following statements:
   1. The sequence \( \{a_{2n}\} \) is in AP with common difference zero.
   2. The sequence \( \{a_{2n+1}\} \) is in AP with common difference zero.

Which of the above statements is/are correct?
   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2

39. What is \( a_{n-1} - a_{n-4} \) equal to?
   (a) -1
   (b) 0
   (c) 1
   (d) 2
For the next two (2) items that follow:

Consider the equation \( x + |y| = 2y \).

40. Which of the following statements are not correct?

1. \( y \) as a function of \( x \) is not defined for all real \( x \).
2. \( y \) as a function of \( x \) is not continuous at \( x = 0 \).
3. \( y \) as a function of \( x \) is differentiable for all \( x \).

Select the correct answer using the code given below.
(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

41. What is the derivative of \( y \) as a function of \( x \) with respect to \( x \) for \( x < 0 \)?
(a) \( \frac{2}{3} \)
(b) \( \frac{1}{3} \)
(c) \( \frac{1}{2} \)
(d) \( \frac{1}{3} \)

For the next two (2) items that follow:

Consider the lines \( y = 3x, y = 6x \) and \( y = 9 \).

42. What is the area of the triangle formed by these lines?

(a) \( \frac{27}{4} \) square units
(b) \( \frac{27}{2} \) square units
(c) \( \frac{19}{4} \) square units
(d) \( \frac{19}{2} \) square units

43. The centroid of the triangle is at which one of the following points?

(a) \( (3, 6) \)
(b) \( \left(\frac{3}{2}, 6\right) \)
(c) \( (3, 3) \)
(d) \( \left(\frac{3}{2}, 9\right) \)

44. What is the number of points of local minima of the function \( f(x) \)?

(a) None
(b) One
(c) Two
(d) Three

45. What is the number of points of local maxima of the function \( f(x) \)?

(a) None
(b) One
(c) Two
(d) Three
46. Suppose $\omega$ is a cube root of unity with $\omega \neq 1$. Suppose $P$ and $Q$ are the points on the complex plane defined by $\omega$ and $\omega^2$. If $O$ is the origin, then what is the angle between $OP$ and $OQ$?

(a) 60°
(b) 90°
(c) 120°
(d) 150°

47. Suppose there is a relation $*$ between the positive numbers $x$ and $y$ given by $x * y$ if and only if $x \leq y^2$. Then which one of the following is correct?

(a) $*$ is reflexive but not transitive and symmetric
(b) $*$ is transitive but not reflexive and symmetric
(c) $*$ is symmetric and reflexive but not transitive
(d) $*$ is symmetric but not reflexive and transitive

48. If $x^2 - px + 4 > 0$ for all real values of $x$, then which one of the following is correct?

(a) $|p| < 4$
(b) $|p| \leq 4$
(c) $|p| > 4$
(d) $|p| \geq 4$

49. If $z = x + iy = \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^{25}$, where $i = \sqrt{-1}$, then what is the fundamental amplitude of $\frac{z - \sqrt{2}}{z - i\sqrt{2}}$?

(a) \(\pi\)
(b) \(\frac{\pi}{2}\)
(c) \(\frac{\pi}{3}\)
(d) \(\frac{\pi}{4}\)

50. If

\[f(x_1) - f(x_2) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)\]

for $x_1, x_2 \in (-1, 1)$, then what is $f(x)$ equal to?

(a) $\ln(1-x)$
(b) $\ln\left(\frac{2+x}{1-x}\right)$
(c) $\tan^{-1}\left(\frac{1-x}{1+x}\right)$
(d) $\tan^{-1}\left(\frac{1+x}{1-x}\right)$

51. What is the range of the function

\[y = \frac{x^2}{1 + x^2}\]

where $x \in \mathbb{R}$?

(a) [0, 1]
(b) [0, 1]
(c) (0, 1)
(d) (0, 1]
52. A straight line intersects $x$ and $y$ axes at $P$ and $Q$ respectively. If $(3, 5)$ is the middle point of $PQ$, then what is the area of the triangle $OPQ$?

(a) 12 square units  
(b) 15 square units  
(c) 20 square units  
(d) 30 square units

53. If a circle of radius $b$ units with centre at $(0, b)$ touches the line $y = x - \sqrt{2}$, then what is the value of $b$?

(a) $2 + \sqrt{2}$  
(b) $2 - \sqrt{2}$  
(c) $\frac{2}{\sqrt{2}}$  
(d) $\sqrt{2}$

54. What is the maximum value of the function $f(\theta)$?

(a) 1  
(b) 2  
(c) 3  
(d) 4

55. What is the minimum value of the function $f(\theta)$?

(a) 0  
(b) 1  
(c) 2  
(d) 3

56. Consider the following statements:

1. $f(\theta) = 2$ has no solution.
2. $f(\theta) = \frac{7}{2}$ has a solution.

Which of the above statements is/are correct?

(a) 1 only  
(b) 2 only  
(c) Both 1 and 2  
(d) Neither 1 nor 2

57. Where do the curves intersect?

Consider the function $f(x) = x|x|-1$ and $g(x) = \begin{cases} \frac{3x}{2}, & x > 0 \\ 2x, & x \leq 0 \end{cases}$

(a) At $(2, 3)$ only  
(b) At $(-1, -2)$ only  
(c) At $(2, 3)$ and $(-1, -2)$  
(d) Neither at $(2, 3)$ nor at $(-1, -2)$
58. What is the area bounded by the curves?

(a) \( \frac{17}{6} \) square units
(b) \( \frac{8}{3} \) square units
(c) 2 square units
(d) \( \frac{1}{3} \) square unit

59. How many solutions does the function \( f(x) = 1 \) have?

(a) One
(b) Two
(c) Three
(d) Four

60. How many solutions does the function \( f(x) = -1 \) have?

(a) One
(b) Two
(c) Three
(d) Four

61. A fair coin is tossed 100 times. What is the probability of getting tails an odd number of times?

(a) \( \frac{1}{2} \)
(b) \( \frac{3}{8} \)
(c) \( \frac{1}{4} \)
(d) \( \frac{1}{8} \)

62. What is the number of ways in which 3 holiday travel tickets are to be given to 10 employees of an organization, if each employee is eligible for any one or more of the tickets?

(a) 60
(b) 120
(c) 500
(d) 1000

63. If one root of the equation \( (l-m)x^2 + lx + 1 = 0 \) is double the other and \( l \) is real, then what is the greatest value of \( m \)?

(a) \( -\frac{9}{8} \)
(b) \( \frac{9}{8} \)
(c) \( -\frac{8}{9} \)
(d) \( \frac{8}{9} \)

64. What is the number of four-digit decimal numbers \(< 1\) in which no digit is repeated?

(a) 3024
(b) 4536
(c) 5040
(d) None of the above
65. What is a vector of unit length orthogonal to both the vectors \( \hat{i} + \hat{j} + \hat{k} \) and \( 2\hat{i} + 3\hat{j} - \hat{k} \)?

(a) \( \frac{-4\hat{i} + 3\hat{j} - \hat{k}}{\sqrt{26}} \)

(b) \( \frac{-4\hat{i} + 3\hat{j} + \hat{k}}{\sqrt{26}} \)

(c) \( \frac{-3\hat{i} + 2\hat{j} - \hat{k}}{\sqrt{14}} \)

(d) \( \frac{-3\hat{i} + 2\hat{j} + \hat{k}}{\sqrt{14}} \)

66. If \( \vec{a}, \vec{b} \) and \( \vec{c} \) are the position vectors of the vertices of an equilateral triangle whose orthocentre is at the origin, then which one of the following is correct?

(a) \( \vec{a} + \vec{b} + \vec{c} = 0 \)

(b) \( \vec{a} + \vec{b} + \vec{c} = \text{unit vector} \)

(c) \( \vec{a} + \vec{b} = \vec{c} \)

(d) \( \vec{a} = \vec{b} + \vec{c} \)

67. What is the area of the parallelogram having diagonals \( 3\hat{i} + \hat{j} - 2\hat{k} \) and \( \hat{i} - 3\hat{j} + 4\hat{k} \)?

(a) \( 5\sqrt{5} \) square units

(b) \( 4\sqrt{5} \) square units

(c) \( 5\sqrt{3} \) square units

(d) \( 15\sqrt{2} \) square units

68. Consider the following in respect of the matrix \( A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \):

1. \( A^2 = -A \)

2. \( A^3 = 4A \)

Which of the above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

69. Which of the following determinants have value ‘zero’?

1. \( \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \)

2. \( \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \)

3. \( \begin{vmatrix} 1 & 2 \\ -2 & -1 \end{vmatrix} \)

Select the correct answer using the code given below.

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3
70. What is the acute angle between the lines represented by the equations \( y - \sqrt{3}x - 5 = 0 \) and \( \sqrt{3}y - x + 6 = 0 \)?

(a) \( 30^\circ \)
(b) \( 45^\circ \)
(c) \( 60^\circ \)
(d) \( 75^\circ \)

71. The system of linear equations

\[ kx + y + z = 1, \quad x + ky + z = 1 \]

and

\[ x + y + kz = 1 \]

has a unique solution under which one of the following conditions?

(a) \( k \neq 1 \) and \( k \neq -2 \)
(b) \( k = 1 \) and \( k \neq 2 \)
(c) \( k \neq -1 \) and \( k \neq -2 \)
(d) \( k \neq -1 \) and \( k \neq 2 \)

72. What is the number of different messages that can be represented by three 0's and two 1's?

(a) 10
(b) 9
(c) 8
(d) 7

73. If \( \log_a (ab) = x \), then what is \( \log_b (ab) \) equal to?

(a) \( \frac{1}{x} \)
(b) \( \frac{x}{x+1} \)
(c) \( \frac{x}{1-x} \)
(d) \( \frac{2}{x} \)

74. If

\[ y = \log_{10} x + \log_x 10 + \log_x x + \log_{10} 10 \]

then what is \( \left( \frac{dy}{dx} \right)_{x=10} \) equal to?

(a) \( 10 \)
(b) \( 2 \)
(c) \( 1 \)
(d) \( 0 \)

75. Suppose \( \omega_1 \) and \( \omega_2 \) are two distinct cube roots of unity different from 1. Then what is \( (\omega_1 - \omega_2)^2 \) equal to?

(a) 3
(b) 1
(c) -1
(d) -3
76. What is the mean deviation from the mean of the numbers 10, 9, 21, 16, 24? 
(a) 5.2
(b) 5.0
(c) 4.5
(d) 4.0

77. Three dice are thrown simultaneously. What is the probability that the sum on the three faces is at least 5? 
(a) \( \frac{17}{18} \)  
(b) \( \frac{53}{54} \)  
(c) \( \frac{103}{108} \)  
(d) \( \frac{215}{216} \)

78. Two independent events A and B have \( P(A) = \frac{1}{3} \) and \( P(B) = \frac{3}{4} \). What is the probability that exactly one of the two events A or B occurs? 
(a) \( \frac{1}{4} \)  
(b) \( \frac{5}{6} \)  
(c) \( \frac{5}{12} \)  
(d) \( \frac{7}{12} \)

79. A coin is tossed three times. What is the probability of getting head and tail alternately? 
(a) \( \frac{1}{8} \)  
(b) \( \frac{1}{4} \)  
(c) \( \frac{1}{2} \)  
(d) \( \frac{3}{4} \)

80. If the total number of observations is 20, \( \Sigma x_i = 1000 \) and \( \Sigma x_i^2 = 84000 \), then what is the variance of the distribution? 
(a) 1500  
(b) 1600  
(c) 1700  
(d) 1800

81. A card is drawn from a well-shuffled deck of 52 cards. What is the probability that it is queen of spade? 
(a) \( \frac{1}{52} \)  
(b) \( \frac{1}{13} \)  
(c) \( \frac{1}{4} \)  
(d) \( \frac{1}{8} \)
82. If two dice are thrown, then what is the probability that the sum on the two faces is greater than or equal to 4?

(a) \[ \frac{13}{18} \]
(b) \[ \frac{5}{6} \]
(c) \[ \frac{11}{12} \]
(d) \[ \frac{35}{36} \]

83. A certain type of missile hits the target with probability \( p = 0.3 \). What is the least number of missiles should be fired so that there is at least an 80% probability that the target is hit?

(a) 5
(b) 6
(c) 7
(d) None of the above

84. For two mutually exclusive events \( A \) and \( B \), \( P(A) = 0.2 \) and \( P(\overline{A} \cap B) = 0.3 \). What is \( P(A | (A \cup B)) \) equal to?

(a) \[ \frac{1}{2} \]
(b) \[ \frac{2}{5} \]
(c) \[ \frac{2}{7} \]
(d) \[ \frac{2}{3} \]

85. What is the probability of 5 Sundays in the month of December?

(a) \[ \frac{1}{7} \]
(b) \[ \frac{2}{7} \]
(c) \[ \frac{3}{7} \]
(d) None of the above

86. If \( m \) is the geometric mean of \( \left( \frac{y}{z} \right)^{\log_{10}(y/z)} \), \( \left( \frac{z}{x} \right)^{\log_{10}(z/x)} \), and \( \left( \frac{x}{y} \right)^{\log_{10}(x/y)} \), then what is the value of \( m \)?

(a) 1
(b) 3
(c) 6
(d) 9

87. A point is chosen at random inside a rectangle measuring 6 inches by 5 inches. What is the probability that the randomly selected point is at least one inch from the edge of the rectangle?

(a) \[ \frac{2}{3} \]
(b) \[ \frac{1}{3} \]
(c) \[ \frac{1}{4} \]
(d) \[ \frac{2}{5} \]
88. The mean of the series \( x_1, x_2, \ldots, x_n \) is \( \bar{x} \). If \( x_2 \) is replaced by \( \lambda \), then what is the new mean?

(a) \( \bar{x} - x_2 + \lambda \)
(b) \( \frac{\bar{x} - x_2 - \lambda}{n} \)
(c) \( \frac{\bar{x} - x_2 + \lambda}{n} \)
(d) \( \frac{n\bar{x} - x_2 + \lambda}{n} \)

89. For the data

3, 5, 1, 6, 5, 9, 5, 2, 8, 6

the mean, median and mode are \( x, y \) and \( z \) respectively. Which one of the following is correct?

(a) \( x = y \neq z \)
(b) \( x \neq y = z \)
(c) \( x = y \neq z \)
(d) \( x = y = z \)

90. Consider the following statements in respect of a histogram:

1. The total area of the rectangles in a histogram is equal to the total area bounded by the corresponding frequency polygon and the \( x \)-axis.
2. When class intervals are unequal in a frequency distribution, the area of the rectangle is proportional to the frequency.

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

91. Consider the following:

1. There exists \( \theta \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right) \) for which
   \[
   \tan^{-1}(\tan(\theta)) \neq \theta.
   \]

2. \[
   \sin^{-1}\left(\frac{1}{3}\right) - \sin^{-1}\left(\frac{1}{5}\right)
   = \sin^{-1}\left(\frac{2\sqrt{2}(\sqrt{3} - 1)}{15}\right)
   \]

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

92. Consider the following statements:

1. \[
   \tan^{-1}\left(\frac{1}{x}\right) + \tan^{-1}\left(\frac{1}{x}\right) = \pi
   \]

2. There exist \( x, y \in [-1, 1] \), where \( x \neq y \) such that

\[
\sin^{-1}x + \cos^{-1}y = \frac{\pi}{2}
\]

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2
93. What are the order and degree respectively of the differential equation whose solution is \( y = cx + c^2 - 3c^{3/2} + 2 \), where \( c \) is a parameter?

(a) 1, 2
(b) 2, 2
(c) 1, 3
(d) 1, 4

94. What is
\[
\int_{-2}^{2} x \, dx - \int_{-2}^{2} [x] \, dx
\]
equal to, where \([x]\) is the greatest integer function?

(a) 0
(b) 1
(c) 2
(d) 4

95. If
\[
\int_{-2}^{5} f(x) \, dx = 4 \quad \text{and} \quad \int_{0}^{5} (1 + f(x)) \, dx = 7
\]
then what is \( \int_{-2}^{5} f(x) \, dx \) equal to?

(a) -3
(b) -2
(c) 3
(d) 5

96. If \( \lim_{x \to 0} \phi(x) = a^2 \), where \( a \neq 0 \), then
what is \( \lim_{x \to 0} \phi\left(\frac{x}{a}\right) \) equal to?

(a) \( a^2 \)
(b) \( a^{-2} \)
(c) \( -a^2 \)
(d) \( -a \)

97. What is \( \lim_{x \to 0} e^{-x^2} \) equal to?

(a) 0
(b) 1
(c) -1
(d) Limit does not exist

98. If \( A \) is a square matrix, then what is \( \text{adj}(A^{-1}) - (\text{adj}A)^{-1} \) equal to?

(a) \( 2 |A| \)
(b) Null matrix
(c) Unit matrix
(d) None of the above

99. What is the binary equivalent of the decimal number 0.3125?

(a) 0.0111
(b) 0.1010
(c) 0.0101
(d) 0.1101

100. Let \( R \) be a relation on the set \( \mathbb{N} \) of natural numbers defined by \( n R m \iff n \) is a factor of \( m \). Then which one of the following is correct?

(a) \( R \) is reflexive, symmetric but not transitive
(b) \( R \) is transitive, symmetric but not reflexive
(c) \( R \) is reflexive, transitive but not symmetric
(d) \( R \) is an equivalence relation
101. What is $\int_{0}^{\pi} |\cos x| \, dx$ equal to?

(a) 0
(b) 2
(c) 4
(d) 8

102. What is the number of natural numbers less than or equal to 1000 which are neither divisible by 10 nor 15 nor 25?

(a) 860
(b) 854
(c) 840
(d) 824

103. $(a, 2b)$ is the mid-point of the line segment joining the points $(10, -6)$ and $(k, 4)$. If $a - 2b = 7$, then what is the value of $k$?

(a) 2
(b) 3
(c) 4
(d) 5

104. Consider the following statements:

1. If $ABC$ is an equilateral triangle, then $\tan (A + B) \tan C = 1$.

2. If $ABC$ is a triangle in which $A = 78^\circ$, $B = 66^\circ$, then
$$\tan \left( \frac{A + C}{2} \right) < \tan A$$

3. If $ABC$ is any triangle, then
$$\tan \left( \frac{A + B}{2} \right) \sin \left( \frac{C}{2} \right) < \cos \left( \frac{C}{2} \right)$$

Which of the above statements is/are correct?

(a) 1 only
(b) 2 only
(c) 1 and 2
(d) 2 and 3

105. If $A = (\cos 12^\circ - \cos 36^\circ)(\sin 96^\circ + \sin 24^\circ)$ and $B = (\sin 60^\circ - \sin 12^\circ)(\cos 48^\circ - \cos 72^\circ)$, then what is $\frac{A}{B}$ equal to?

(a) -1
(b) 0
(c) 1
(d) 2
For the next four (4) items that follow:

Let \( f : \mathbb{R} \rightarrow \mathbb{R} \) be a function such that
\[
f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)
\]
for \( x \in \mathbb{R} \).

106. What is \( f(1) \) equal to?

(a) -2
(b) -1
(c) 0
(d) 4

107. What is \( f'(1) \) equal to?

(a) -6
(b) -5
(c) 1
(d) 0

108. What is \( f'''(10) \) equal to?

(a) 1
(b) 5
(c) 6
(d) 8

109. Consider the following:

1. \( f(2) = f(1) - f'(0) \)
2. \( f''(2) - 2f'(1) = 12 \)

Which of the above is/are correct?

(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

For the next three (3) items that follow:

A plane \( P \) passes through the line of intersection of the planes

\[2x - y + 3z = 2, \quad x + y - z = 1\]

and the point \((1, 0, 1)\).

110. What are the direction ratios of the line of intersection of the given planes?

(a) \( \{2, -5, -3\} \)
(b) \( \{1, -5, -3\} \)
(c) \( \{2, 5, 3\} \)
(d) \( \{1, 3, 5\} \)
111. What is the equation of the plane $P$?

(a) $2x + 5y - 2 = 0$
(b) $5x + 2y - 5 = 0$
(c) $x + z - 2 = 0$
(d) $2x - y - 2z = 0$

112. If the plane $P$ touches the sphere $x^2 + y^2 + z^2 = r^2$, then what is $r$ equal to?

(a) $\frac{2}{\sqrt{29}}$
(b) $\frac{4}{\sqrt{29}}$
(c) $\frac{5}{\sqrt{29}}$
(d) 1

113. What is $f''(4)$ equal to?

(a) $-4$
(b) $-3$
(c) 3
(d) 2

114. What is $f''(2.5)$ equal to?

(a) $-3$
(b) $-2$
(c) 0
(d) 2

For the next two (2) items that follow:

Let $f(x)$ be the greatest integer function, and $g(x)$ be the modulus function.

115. What is $(f \circ f)(-\frac{5}{3}) - (f \circ g)(-\frac{5}{3})$ equal to?

(a) $-1$
(b) 0
(c) 1
(d) 2

116. What is $(f \circ f)(-\frac{9}{5}) + (g \circ g)(-2)$ equal to?

(a) $-1$
(b) 0
(c) 1
(d) 2
For the next two (2) items that follow:

Consider a circle passing through the origin and the points \((a, b)\) and \((-b, -a)\).

117. On which line does the centre of the circle lie?
   
   \( (a)\) \(x + y = 0\)
   
   \( (b)\) \(x - y = 0\)
   
   \( (c)\) \(x + y = a^2 + b^2\)
   
   \( (d)\) \(x - y = a^2 - b^2\)

118. What is the sum of the squares of the intercepts cut off by the circle on the axes?

\( (a)\) \(\left(\frac{a^2 + b^2}{a^2 - b^2}\right)^2\)

\( (b)\) \(2\left(\frac{a^2 + b^2}{a - b}\right)^2\)

\( (c)\) \(4\left(\frac{a^2 + b^2}{a - b}\right)^2\)

\( (d)\) None of the above

For the next two (2) items that follows:

Let \(\hat{a}, \hat{b}\) be two unit vectors and \(\theta\) be the angle between them.

119. What is \(\cos\left(\frac{\theta}{2}\right)\) equal to?

\( (a)\) \(\frac{|\hat{a} - \hat{b}|}{2}\)

\( (b)\) \(\frac{|\hat{a} + \hat{b}|}{2}\)

\( (c)\) \(\frac{|\hat{a} - \hat{b}|}{4}\)

\( (d)\) \(\frac{|\hat{a} + \hat{b}|}{4}\)

120. What is \(\sin\left(\frac{\theta}{2}\right)\) equal to?

\( (a)\) \(\frac{|\hat{a} - \hat{b}|}{2}\)

\( (b)\) \(\frac{|\hat{a} + \hat{b}|}{2}\)

\( (c)\) \(\frac{|\hat{a} - \hat{b}|}{4}\)

\( (d)\) \(\frac{|\hat{a} + \hat{b}|}{4}\)